**Negative Mass Dark Matter Gravitational Bound States and the Formation of “Dark Voids”: A Topological Condensation Model Based on ABC Theory**

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Abstract:  
Based on Li Zhijun’s ABC theory, this paper systematically proposes, for the first time, a theoretical model wherein negative mass dark matter (NMDM) forms macroscopic quantum bound states—“Dark Voids”—under specific conditions. Unlike black holes formed from ordinary matter (positive mass, gravitational attraction), a Dark Void is formed from negative mass dark matter particles through a self-organizing mechanism of their repulsive gravity, manifesting as a stable, low-density vacuum region with macroscopic quantum properties. The core thesis is: on cosmological scales, negative mass dark matter can undergo a topological phase transition via its specific coupling with the Higgs field vacuum, forming an anti-bound state. The energy density of this state is lower than the average of the surrounding spacetime, creating a gravitational potential well (but with the opposite sign). We construct modified Einstein field equations describing the gravitational interaction of negative mass matter, derive the critical conditions for Dark Void formation, provide a topological criterion for their stability, and calculate their unique gravitational lensing and cosmological signals. This model predicts a novel class of celestial objects, providing a testable observational window for detecting negative mass dark matter.

Keywords: Negative mass dark matter; Dark Void; Topological condensation; Repulsive gravity; Modified field equations; ABC theory; Cosmological probe

1. **Introduction: The Cosmological Conundrum of Negative Mass Dark Matter and a New Paradigm**

1.1 Properties of Negative Mass Dark Matter

According to Li Zhijun’s ABC theory, the sole distinction between negative mass dark matter (NMDM) and ordinary matter (OM) lies in the sign of the Higgs field vacuum to which they couple:

* Ordinary Matter (OM): Couples to the vacuum, , produces attractive gravity.
* Negative Mass Dark Matter (NMDM): Couples to the vacuum, , produces repulsive gravity (its stress-energy tensor causes geodesic deviation).

1.2 Physical Intuition for “Dark Void” Formation

Positive mass matter collapses under its own gravity to form black holes (high density, strong attraction). How, then, would negative mass matter evolve under its own “self-repulsion”?

Intuitively, repulsion should lead to material diffusion. However, under certain conditions, a self-repelling system can reach an equilibrium through interaction with background fields, forming a stable, low-density structure. This is analogous to the spatial homogenization caused by bosonic repulsive interactions but occurring at the gravitational level.

We term this stable state formed by NMDM, with a clear boundary and internal structure, a “Dark Void”.

1. **Theoretical Framework: Gravitation and Field Equations for Negative Mass Matter**

2.1 Modified Einstein Field Equations

When negative mass dark matter is included, the total stress-energy tensor is:

where the trace

The Einstein field equations remain unchanged:

However, due to the peculiar sign of the components in unique geometric effects arise.

2.2 Hydrodynamic Equations for Negative Mass Matter

Treating NMDM as an ideal fluid, its energy-momentum tensor is:

where The equation of state is assumed to be

From energy-momentum conservation in the Newtonian limit, we obtain a modified Poisson equation:

Since the NMDM contributes a negative source term for the gravitational potential, effectively producing a repulsive force.

2.3 Equilibrium Conditions for a Dark Void

For a spherically symmetric dark matter cluster, its internal pressure resists its own “self-repulsion” to achieve equilibrium. The hydrostatic equilibrium equation is:

Since and self-repulsion leads to (increasing outward), the right-hand side of the equation is negative, indicating that pressure decreases from the inside out. This is opposite to an ordinary star.

1. **Topological Condensation Mechanism for Dark Void Formation**

3.1 Field Combination and Phase Transition Conditions

The field combination for an NMDM particle is:

Its mass is

Forming a Dark Void requires NMDM to achieve macroscopic quantum coherence, i.e., its color charge field B component forms a Bose-Einstein condensate (BEC). This requires:  
1. Phase space density condition: where is the thermal de Broglie wavelength.  
2. Interaction condition: Sufficient interaction between particles to establish long-range coherence.

For NMDM, the effective interaction is dominated by the exchange of virtual gravitons.

3.2 Effective Theory of the Condensate

After condensation, the system can be described by a Gross-Pitaevskii equation, but the gravitational potential is determined by its own density distribution:

where is the wave function of the dark matter condensate, is the self-interaction coefficient, and is the gravitational potential, satisfying:

Note that so the sign of the term must be handled carefully.

3.3 Topological Stability and Boundary Conditions

The stability of a Dark Void relies on its boundary. We assume a sharp interface exists, outside of which is the ordinary vacuum or low-density dark matter, and inside is the NMDM condensate.

At the boundary, the wave function tends to zero, and the gravitational potential and its derivative are continuous. This boundary is stabilized by topological defects (e.g., vortices), characterized by a winding number:

A non-zero guarantees the stability of the state.

1. **Static Solutions and Properties of Dark Voids**

4.1 Non-dimensionalization and Numerical Solution

Introduce dimensionless variables:

where is the healing length, is the central density, and is the central potential.

The governing equations become:

where .

Numerical solving yields the density distribution and gravitational potential distribution of the Dark Void.

4.2 Macroscopic Properties

* Low-density core: The density inside the Dark Void is much lower than the cosmic average density.
* Gravitational potential hill: Due to its repulsive nature, the center of the Dark Void is a gravitational potential “hill” (), not a well.
* Sharp boundary: The density drops sharply to zero at a finite radius
* Mass-radius relation: The total (negative) mass and radius relationship differs from that of ordinary celestial objects.

1. **Cosmological Observational Signals**

5.1 Gravitational Lensing Effect

Due to the low internal density and positive gravitational potential of a Dark Void, light passing near it diverges rather than converges, producing a negative gravitational lensing effect:

where is the impact parameter. This creates faint dilution rings around background galaxy images instead of Einstein rings.

5.2 Cosmic Microwave Background (CMB) Anisotropy

Dark Voids would cause a blueshift of CMB photons in their vicinity (as photons gain energy falling into a positive potential well):

This manifests as local hot spots on the CMB map.

5.3 Large-Scale Structure Formation

As low-density regions, Dark Voids would inhibit the clustering of matter around them, leaving vast voids in the cosmic web, potentially much larger than those formed by positive mass matter.

1. **Stability Analysis**

6.1 Linear Stability

Applying small perturbations to the equilibrium state yields linearized GP-Poisson equations. Analyzing the eigenmode frequencies :

* If all , it is stable.
* If any , it is unstable.

Due to the repulsive interaction, a stable branch is expected to exist.

6.2 Topological Stability

The topological charge provides topological protection. Changing requires crossing a large energy barrier, making topologically stable Dark Voids long-lived.

1. **Conclusion and Outlook**

This paper presents a theoretical model for the formation of “Dark Voids” from negative mass dark matter:

1. Formation mechanism: NMDM undergoes a topological phase transition to form a macroscopic quantum condensate.
2. Equilibrium state: Self-repulsion balances the pressure gradient, forming a low-density structure with a sharp boundary.
3. Observational signals: Negative gravitational lensing, CMB hot spots, suppression of structure formation.
4. Stability: Protected by topological charge.

**Future work:**

1. Solve the GP-Poisson equations precisely.
2. Calculate specific observational predictions (lensing signals, void statistics).
3. Explore the relationship between Dark Voids and the cosmological constant.

This model opens new avenues for detecting negative mass dark matter and demonstrates the predictive power of ABC theory.

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